Dr. Mark Pokras: Solutions

Problem #1

Create a spreadsheet whose columns represent the number of deer of a given age, and whose rows represent the number of years that have passed.

Since we start with 2-year-old deer, and we are going to look ten years into the future, we will set the table up with columns for deer up to age 12.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	year	0 yr-olds	1 yr-olds	2 yr-olds	3 yr-olds	4 yr-olds	5 yr-olds	6 yr-olds	7 yr-olds	8 yr-olds	9 yr-olds	10 yr-olds	11 yr-olds	12 yr-olds	total
2	0	0	0	16	0	0	0	0	0	0	0	0	0	0	16

The entries at the beginning of the period (year = 0) are shown above: 16 deer, all 2 years old.

After 1 year (year = 1), the original 16 deer have become one year older. Thus the value for cell E3 is the same as the value for cell D2. With a little thought, it is clear that this pattern will be repeated throughout the spreadsheet: the value for a cell will be equal to the value of the cell that is up one row and one column to the left of it.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	year	0 yr-olds	1 yr-olds	2 yr-olds	3 yr-olds	4 yr-olds	5 yr-olds	6 yr-olds	7 yr-olds	8 yr-olds	9 yr-olds	10 yr-olds	11 yr-olds	12 yr-olds	total
2	0	0	0	16	0	0	0	0	0	0	0	0	0	0	16
3	1				16										

However, this does not work for the cells in column A: the number of deer that are "zero" years old, that is, the fawns born each year. Consider the value of cell A3: the number of fawns born after one year. According to the given data, there are <u>two</u> fawns born for every <u>pair</u> of deer over two years old, or <u>one</u> fawn for <u>each</u> single deer. At year zero, there were 16 deer over two years, old, thus there should be 16 fawns born in year 1. There are no deer of any other age, so the total number of deer is the fawns (16) plus the three-year olds (16), or 32 deer.

After one year the spreadsheet looks like this:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1		0 yr-olds	1 yr-olds	2 yr-olds	3 yr-olds	4 yr-olds	5 yr-olds	6 yr-olds	7 yr-olds	8 yr-olds	9 yr-olds	10 yr-olds	11 yr-olds	12 yr-olds	Total
2	0	0	0	16	0	0	0	0	0	0	0	0	0	0	16
3	1	16	0	0	16	0	0	0	0	0	0	0	0	0	32

As year 1 ends and year 2 begins, another 16 fawns will be born, and every other deer will get one year older. The same thing happens at the beginning of year 3.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1		0 yr-olds	1 yr-olds	2 yr-olds	3 yr-olds	4 yr-olds	5 yr-olds	6 yr-olds	7 yr-olds	8 yr-olds	9 yr-olds	10 yr-olds	11 yr-olds	12 yr-olds	Total
2	0	0	0	16	0	0	0	0	0	0	0	0	0	0	16
3	1	16	0	0	16	0	0	0	0	0	0	0	0	0	32
4	2	16	16	0	0	16	0	0	0	0	0	0	0	0	48
5	3	16	16	16	0	0	16	0	0	0	0	0	0	0	64

Notice that at this point in time (after three years have passed), the fawns that were born by the end of the first year are now themselves 2 years old. Thus they will begin to reproduce. Altogether at this point we have 32 deer of age two or older, and thus 32 fawns will be born the following year (beginning of year 4):

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	Year	0 yr-olds	1 yr-olds	2 yr-olds	3 yr-olds	4 yr-olds	5 yr-olds	6 yr-olds	7 yr-olds	8 yr-olds	9 yr-olds	10 yr-olds	11 yr-olds	12 yr-olds	total
1	0	0	0	16	0	0	0	0	0	0	0	0	0	0	16
2	1	16	0	0	16	0	0	0	0	0	0	0	0	0	32
3	2	16	16	0	0	16	0	0	0	0	0	0	0	0	48
4	3	16	16	16	0	0	16	0	0	0	0	0	0	0	64
5	4	32	16	16	16	0	0	16	0	0	0	0	0	0	96

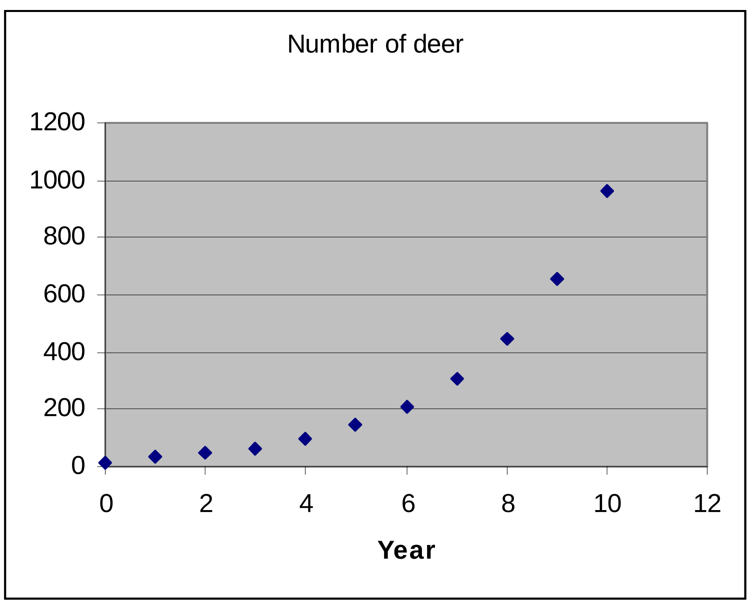
Thinking about this, we realize that the number of fawns born each year is equal to the number of deer that are age 2 and older in the prior year. This can be entered as a formula in the “0-year-olds” column: the sum of deer in the row above, starting at the two-year-old column and moving to the right.

Now we have a formula for the cells in the "0-year-olds" column (the sum of cells one row above, starting at column D), and for every other cell (the value of the cell one row up and one column over). We can paste these formulas into the spreadsheet up to year 12, with this result:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Year	0 yr-olds	1 yr-olds	2 yr-olds	3 yr-olds	4 yr-olds	5 yr-olds	6 yr-olds	7 yr-olds	8 yr-olds	9 yr-olds	10 yr-olds	11 yr-olds	12 yr-olds	total
2	0	0	0	16	0	0	0	0	0	0	0	0	0	0	16
3	1	16	0	0	16	0	0	0	0	0	0	0	0	0	32
4	2	16	16	0	0	16	0	0	0	0	0	0	0	0	48
5	3	16	16	16	0	0	16	0	0	0	0	0	0	0	64
6	4	32	16	16	16	0	0	16	0	0	0	0	0	0	96
7	5	48	32	16	16	16	0	0	16	0	0	0	0	0	144
8	6	64	48	32	16	16	16	0	0	16	0	0	0	0	208
9	7	96	64	48	32	16	16	16	0	0	16	0	0	0	304
10	8	144	96	64	48	32	16	16	16	0	0	16	0	0	448
11	9	208	144	96	64	48	32	16	16	16	0	0	16	0	656
12	10	304	208	144	96	64	48	32	16	16	16	0	0	16	960

Problem #2

The plotted graph does look like it might be an exponential function:



The general equation of an exponential function is $y = Ae^{(bx)}$.

To determine the specific equation for this situation, we need to find the values of "A" and "b".

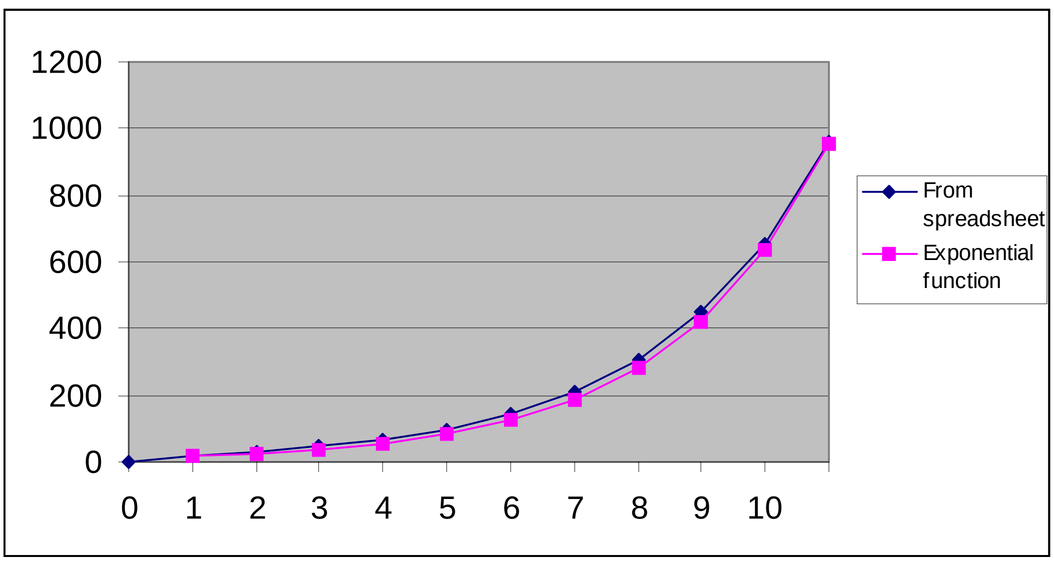
When $x = 0$ (time is zero), the number of deer is 16, giving A = 16. To find the value of "b", we can plug in the values of x and y after ten years (x = 10, y = 960):

$$960 = 16*e^{(10*b)}$$
$$10*b = \ln(960/16) = \ln(60)$$
$$b = 0.1*\ln(60) = 0.409$$

So our exponential function would be

$$y = 16e^{(.409x)}$$

This function matches the spreadsheet result quite closely:



This model may not be accurate, since it does not take into account the fact that the deer may run out of food or be slain by predators.